

MARKSCHEME

May 2007

MATHEMATICAL STUDIES

Standard Level

Paper 2

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Paper 2 Markscheme
Instructions to Examiners

Notes: If in doubt about these instructions or any other marking issues, contact your team leader for clarification.

Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

1 Abbreviations

The markscheme may make use of the following abbreviations:

M Marks awarded for **Method**

A Marks awarded for an **Answer** or for **Accuracy**

G Marks awarded for correct solutions obtained from a **Graphic Display Calculator**, irrespective of working shown.

R Marks awarded for clear **Reasoning**

AG **Answer Given** in the question and consequently, marks not awarded.

ft Marks that can be awarded as **follow through** from previous results in the question

In paper 2 candidates are expected to demonstrate their ability to communicate mathematics using appropriate working. Answers which are correct but not supported by adequate working will **not always receive full marks**. Marks to be awarded for unsupported answers are designated **G** in the mark scheme as such answers will usually arise from working performed on a graphic display calculator.

2 Method of Marking

(a) All marking must be done using a **red** pen.

(b) Marks must be noted on candidates' scripts as in the markscheme:

- Show the breakdown of individual marks using the **abbreviations** (**MI**), (**A2**) *etc*;
- Write down each part mark total, as indicated on the **question paper**. These totals should be written in the **margins** of the candidates' answer booklets;
- Write down and circle the total for each question at the end of the question.
- Transfer the total for **each question** to the front cover sheet and write down the total mark for the paper

(c) Working crossed out by the candidate should not be awarded any marks.

(d) Where candidates have written two solutions to a question, only the first solution should be marked.

- (e) If correct working results in a correct answer but then further working is developed, full marks are **not** always awarded. In most such cases it will be a single final answer mark that is lost. Full marks **can** be awarded if the candidate demonstrates clear understanding of the task and the result. If in doubt, consult your team leader.
- (f) Candidate drawn graphs will have a single **(AI)** available for scales and labels combined. This can be awarded if all these are present and correct, even if no graph is drawn, however, the mark should not be awarded if the scale shown is inappropriate to, or inadequate for, the required missing graph. In papers which have two candidate drawn graphs, consistent errors in showing labels or scales can follow through on the second graph, though not if the error is complete omission of these features.

Please note: Assignment of marks to the answers in all the following examples is for demonstration purposes only. Marks for actual examination questions will not necessarily follow the same pattern.

Question: Using Pythagoras to find a side of a triangle:

Markscheme	Candidates' Scripts	Marking
$\sqrt{9+4} = \sqrt{13}$ (MI)(AI) (3.61 3s.f.)	Case (i) $\sqrt{13}$ or 3.61 or both	(G2)
OR Answer only (G2)	Case (ii) $\sqrt{9+4} = \sqrt{13}$ = 6.50	(MI) (A0)

Question: Calculate the gradient of the line passing through the points (5,3) and (0,9).

Markscheme	Candidates' Scripts	Marking
$\frac{9-3}{0-5} = -\frac{6}{5}$ (MI)(AI)	(i) $-6/5$	(G1)
OR Answer only (G1)	(ii) $\frac{9-3}{0-5} = -\frac{6}{5}$ Gradient is $-6/5$ $y = -6x/5 + 9$	(MI) (AI) <i>(There is clear understanding of the gradient.)</i>
	(iii) $\frac{9-3}{0-5} = -\frac{6}{5}$ $y = -6x/5 + 9$	(MI) (A0) <i>(There is confusion about what is required.)</i>

Question: sine rule used to find angle A , with angle B and side b known but side a is first calculated using Pythagoras in an adjoining triangle.

Markscheme		Candidate's Script		Marking
$a = \sqrt{25 + 36} = \sqrt{61}$	(M1)(A1)	Case (i)	$a = \sqrt{61}$	(G2)
OR answer only	(G2)		$A = 55.9^\circ$	(A2)
$\frac{\sin(A)}{\sqrt{61}} = \frac{\sin(32)}{5}$	(M1)(A1)	Case (ii)	$A = 55.9^\circ$ (with no mention of a)	(A2)
$A = 55.9^\circ$	(A1)			
OR answer only	(A2)			

3 Follow-through (ft) Marks

Errors made at any step of a solution can affect all working that follows. To limit the severity of the penalty, **follow through (ft)** marks can be awarded. Markschemes will indicate where it is appropriate to apply follow through in a question with '**ft**' appended to the eligible mark(s).

- If an answer resulting from follow through is extremely unrealistic (e.g. negative distances or wrong by a large order of magnitude) then the final **A** mark should not be awarded. If in doubt, contact your team leader.
- If a question is transformed by an error into a **different, much simpler question** then follow through might not apply or might be reduced. In this situation consult your team leader and record the decision on the candidate's script.
- To award follow through marks for a question part, **there must be working present for that part** and not just an answer based on the follow through. An isolated follow through answer, with no working, must be regarded as incorrect and receives no marks **even if it seems approximately correct**.
- Inadvertent use of radians will be penalised the first time it occurs. Subsequent use, even in later questions will normally be allowed follow through marks unless the answer is unrealistic. Cases of this kind will be addressed on an individual basis.

Implementation: The following examples illustrate correct use of the **follow through** process in straightforward situations.

Question: An investment problem with two different rates of interest and a total amount of \$600 split across the rates in consecutive periods:

Markscheme	Candidate's Script	Marking
(a) $\$ 600 \times 1.02$ $= \$ 612$	Case (i)	
	(a) Final amount after 1 st period	(M1)
	$= \$ 600 \times 1.02$	
OR answer only	$= \$ 602$	(A0)
(b) $\$ \left(\frac{612}{2} \times 1.02 \right) + \left(\frac{612}{2} \times 1.04 \right)$	(b) Amount after 2 nd period	(M1)
	$= 301 \times 1.02 + 301 \times 1.04$	
	$= \$ 620.06$	(A1)(ft)
OR answer only	but note	
Note: The (M1) is for splitting the value from (a) and forming a sum of products.	Case (ii)	
	an (M0) almost always prohibits	
Here the (ft) indicates a possible follow through from part (a).	the associated (ft) so	
	(a) $\$ 600 \times 1.02 = \$ 602$	(M1)(A0)
	(b) $\$ 602 \times 1.04 = \$ 626.08$	(M0)(A0)(ft)
	Case (iii)	
	(a) $\$ 600 \times 1.02 = \$ 602$	(M1)(A0)
	(b) No working. 620.06 given as answer.	(G0)(ft)
	Case (iv)	
	(a) $\$ 612$	(G2)
	(b) $\$ 630.36$	(G1)

Question: Using trigonometry to calculate angles and sides of triangles.

Markscheme		Candidate's Script		Marking	
(a)	$\frac{\sin A}{3} = \frac{\sin 30}{4}$ $A = 22.0^\circ$	(MI)(AI)	(a)	$\frac{\sin A}{4} = \frac{\sin 30}{3}$ $A = 41.8^\circ$	(MI) (A0) <i>(use of sine rule but with wrong values)</i> (A0) <i>(Note: the 2nd (AI) here was not marked (ft) and cannot be awarded, because there was an earlier error in the same question part.)</i>
OR	answer only	(A2)			
(b)	$x = 7 \tan A$ $= 2.83$	(MI) (AI)(ft)	(b)	case (i) $x = 7 \tan A$ $= 6.26$	(MI) (AI)(ft)
OR	answer 2.83 only	(GI)	but case (ii) 6.26		(G0)

4 Using the Markscheme

This markscheme presents a particular way in which each question might be worked and how it should be marked.

- (a) As **A** marks are normally **dependent** on the preceding **M** mark being awarded, it is **not** possible to award **(M0)(AI)**. Once an **(M0)** has been awarded, all subsequent **A** marks are lost in that part of the question, even if calculations are performed correctly, until the next **M** mark, unless otherwise instructed in the markscheme. (See the finance example above).

Similarly **(AI)(R0)** cannot be awarded for an answer which is accidentally correct for the wrong reasons given.

Example: Question: (a) χ^2 calculated followed by (b) degrees of freedom found and (c) and (d) comparison to critical value. (Dependence of **A** and **R** marks.)

Markscheme		Candidate's Script		Marking
(a)	$\chi_{calc}^2 = 3.92$	(AI)	Case (i)	
(b)	$n = 4$	(AI)	(a) $\chi_{calc}^2 = 3.92$	(AI)
(c)	$\chi_{crit}^2 = 9.488$	(AI)(ft)	(b) $n = 4$	(AI)
(d)	Do not reject null hypothesis because $\chi_{calc}^2 < \chi_{crit}^2$	(AI)(ft) (RI)(ft)	(c) Don't know?	(A0)
			(d) Do not reject null hypothesis because $\chi_{calc}^2 > 0$	(A0)(ft) (R0)(ft) <i>((A0) was awarded here because the reason is wrong.)</i>

	Case (ii)	
	(a) $\chi_{calc}^2 = 3.92$	(AI)
	(b) $n = 4$	(AI)
	(c) $\chi_{crit}^2 = 4.488$	(A0)
	(d) Do not reject null hypothesis because $\chi_{calc}^2 < \chi_{crit}^2$	(AI)(ft) (RI)(ft)
	Case (iii)	
	(a) $\chi_{calc}^2 = 3.92$	(AI)
	(b) $n = 1$	(A0)
	(c) $\chi_{crit}^2 = 3.841$	(AI)(ft)
	(d) Reject null hypothesis because $\chi_{calc}^2 > \chi_{crit}^2$	(AI)(ft) (RI)(ft)

- (b) **Alternative methods** have not always been included. Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method in a manner that is consistent with the markscheme.

Where alternative methods for complete questions are included in the markscheme, they are indicated by ‘**OR**’ etc. This includes alternatives obtained with a graphic display calculator. In such cases, alternative **G** mark assignments for answer only will not be repeated if this is redundant.

Example: Question to find the coordinates of a vertex of a given quadratic.

Working	Marks
$f(x) = 2x^2 + 7x - 3$	
$x = -\frac{b}{2a} = -\frac{7}{4}$	(M1)(A1) or (G2)
(M1) for use of $-b/2a$, (A1) for correct answer	
$f(-7/4) = -\frac{146}{16} = -\frac{73}{8}$	(M1)(A1)(ft)
(M1) for using $f(-7/4)$, (A1) for answer.	or (G1)
Coordinates are $(-7/4, -73/8)$	(A1)(ft)
OR	OR

5 Accuracy of Answers

Unless otherwise stated in the question, all numerical answers must be given exactly or correct to 3 significant figures.

A penalty known as an **ACCURACY PENALTY (AP)** is applied if an answer is either

- (i) rounded incorrectly to 3 significant figures or
- (ii) rounded correctly or incorrectly to some other level of accuracy.

This penalty is applied to the **final answer** of a question part only. It applies **also** when an exact answer is incorrectly rounded.

THE ACCURACY PENALTY IS APPLIED AT MOST ONCE PER PAPER! Subsequent accuracy errors can be **ignored** and full marks awarded if all else is correct.

An accuracy penalty must be recorded in proximity to the incorrect answer as $(A0)(AP)$.

Examiners must record the occurrence of an accuracy penalty by writing (AP) next to the relevant question total on the front of the cover sheet.

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. In **all** such cases the final mark is not awarded if the rounding does not follow the instructions given in the question. This is **NOT** an accuracy penalty. A mark for specified accuracy can be regarded as a **(ft)** mark regardless of an immediately preceding $(M0)$.

Rounding of an exact answer to 3 significant figures **should be accepted if performed correctly.** If the rounding is incorrect, an accuracy penalty should be applied as detailed above. Exact answers such as $\frac{1}{4}$ can be written as decimals to less than three significant figures if the result is still exact. Reduction of a fraction to its lowest terms is **not** essential.

Ratios of π and answers taking the form of square roots of integers (**even if exact squares**) or any rational power of an integer (*e.g.* $\sqrt{13}$, $2^{2/3}$, $\sqrt[4]{5}$, $\sqrt{9}$) may be accepted as exact answers. All other powers (*e.g.* of non-integers) and values of transcendental functions such as sine and cosine must be evaluated.

Answers **with no supporting working** which are written correct to more than 3 significant figures should be marked according to the scheme for correct answers with no working, but with an (AP) then applied. When this happens, $(A2)$ or $(G2)$ can be split if necessary (*e.g.* $(A1)(A0)(AP)$ or $(G1)(G0)(AP)$). **Unsupported** answers with less than 3 significant figures must be deemed incorrect even if they seem approximately correct.

An accuracy penalty should not be applied to an answer that is already incorrect for some other reason.

Special cases

An answer taken directly from the IB chi-squared statistical table can be given and used to the same level of accuracy as appears in the table (3 decimal places) or correct to 3 significant figures.

For judging equivalence between 3 significant figures and use of minutes and seconds for angles, guidelines have been issued to paper setters. This problem will be dealt with on an individual basis as the need arises.

Examples: The Pythagoras example used before:

Markscheme	Candidates' Scripts	Marking
$\sqrt{9+4} = \sqrt{13}$ (MI)(AI) (3.61 3s.f.) OR answer only (G2)	(i) 3.6 or 4	(G0)
	(ii) 3.60555	(G1)(G0)(AP)
	(iii) $\sqrt{9+4} = \sqrt{13}$ = 3.6	(M1) (A0)(AP)
	(iv) $\sqrt{9+4} = \sqrt{13}$ = 3.60555	(M1) (A0)(AP)
	(v) $\sqrt{9+4} = \sqrt{13} = 3.60$	(M1)(A0)(AP)
	(vi) $\sqrt{9+4} = \sqrt{14} = 3.74$	(M1)(A0)

If the question specified e.g. correct to 4 decimal places for the answer, then there would be one extra mark available as follows:

Markscheme	Candidates' Scripts	Marking
$\sqrt{9+4} = \sqrt{13}$ (MI)(AI) OR answer only (G2) (Note: requires more than 4 d.p.) = 3.6056 (4 d.p.) (AI)(ft) OR answer only (G2) OR answer 3.606 or 3.61 only (G1)	(i) 3.605551 = 3.6056 (4 d.p.)	(G2)(AI)
	(ii) $\sqrt{9+4} = \sqrt{13}$ = 3.606	(M1)(AI) (A0)
	(iii) 3.60555	(G2)(A0)
	(iv) 3.6056	(G2)
	(v) $\sqrt{9+4} = \sqrt{14}$ = 3.7417	(M1)(A0) (AI)(ft)
	(vi) $\sqrt{9-4} = \sqrt{5}$ = 2.2361	(M0)(A0) (AI)(ft) (Note: this is a special case, where the initial (M0) does not determine the final (A0) because the correction to 4dp is an entirely new task.)
	(vii) 3.606	(G1)

Premature Rounding

Accuracy errors in a final answer, which result from premature rounding earlier in the same question part, should not receive an accuracy penalty. There are two situations. If there is a mark available for a prematurely rounded answer and the rounding occurs at this stage, then the inappropriate rounding should be penalised with *(A0)* but the answer can then be allowed to follow through to the end of the question. If the first stage of the answer is correct but rounded further on, then it should be penalised at an appropriate place close to where it is rounded. Some discretion should be used to deny a *(ft)* mark if the rounding is very bad and the answer far from its required value.

Example: Question: sine rule used to find angle *A*, with angle *B* and side *b* known but side *a* is first calculated using Pythagoras in an adjoining triangle.

Markscheme	Candidate's Script	Marking
$a = \sqrt{25 + 36} = \sqrt{61}$ <i>(M1)(A1)</i> OR answer only <i>(G2)</i>	(i) $a = \sqrt{25 + 36} = \sqrt{61}$ $= 7.8$	<i>(M1)</i> <i>(A0)</i>
$\frac{\sin(A)}{\sqrt{61}} = \frac{\sin(32)}{5}$ <i>(M1)(A1)(ft)</i> $A = 55.9^\circ$ <i>(A1)(ft)</i> OR answer only <i>(G2)</i>	$\frac{\sin(A)}{7.8} = \frac{\sin(32)}{5}$ $A = 55.8^\circ$	<i>(M1)(A1)(ft)</i> <i>(A1)(ft)</i>
	(ii) $a = \sqrt{25 + 36} = \sqrt{61}$ $\frac{\sin(A)}{7.8} = \frac{\sin(32)}{5}$ $A = 55.8^\circ$	<i>(M1)(A1)</i> <i>(M1)(A0)</i> <i>(A1)(ft)</i>
	(iii) $a = \sqrt{25 + 36} = \sqrt{61}$ $\frac{\sin(A)}{7.8} = \frac{\sin(32)}{5}$ $A = 55.9^\circ$	<i>(M1)(A1)</i> <i>(M1)(A0)</i> <i>(A0)(AP)(ft)</i> <i>(even though this is the answer to the question, the rounded answer does not follow from the given working.)</i>
	(iv) $a = \sqrt{25 + 36} = \sqrt{61}$ $\frac{\sin(A)}{7.8} = \frac{\sin(32)}{5}$ $A = \sin^{-1}(0.83) = 56.1^\circ$	<i>(M1)(A1)</i> <i>(M1)(A0)</i> <i>(A0)</i>

	<p>(v) $a = \sqrt{25 + 36} = \sqrt{61} = 8$</p> $\frac{\sin(A)}{8} = \frac{\sin(32)}{5}$ <p>$A = 58.0^\circ$</p>	<p>(M1)(A0)</p> <p>(M1)(A1)(ft)</p> <p>(A0)(ft) <i>(The rounding is severe and the answer quite far from correct).</i></p>
	<p>(vi) $a = 7.8$</p> <p>$A = 55.8^\circ$</p>	<p>(G0)</p> <p>(G0)(ft) <i>(there is no working to justify the follow through.)</i></p>

6 Level of accuracy in finance questions

The accuracy level required for answers will be specified in all questions involving money. This will usually be either whole units or two decimal places, but could differ in rare instances depending on the currency in question.

A penalty known as a FINANCIAL ACCURACY PENALTY (**FP**) is applied if an answer does not adhere to the specification in the question. This penalty is applied to the final answer of a question part only.

THE FINANCIAL ACCURACY PENALTY IS APPLIED AT MOST ONCE PER PAPER!
 Subsequent financial accuracy errors can be ignored and full marks awarded if all else is correct.

A financial accuracy penalty must be recorded in proximity to the incorrect answer as **(A0)(FP)**. Examiners must record the occurrence of a financial accuracy penalty by writing **(FP)** next to the relevant question total on the front cover sheet.

The financial accuracy penalty is imposed only for rounding to the wrong level of accuracy and NOT for incorrect rounding to the required number of places. The latter would incur a normal accuracy penalty (**AP**). **No single answer can receive two penalties. If both types of error are present then (FP) takes priority.**

Please see the examples below.

NOTE: The financial accuracy penalty will be flagged in the markscheme at the start of each answer where it could apply, with the words “Financial accuracy penalty (**FP**) is applicable in this question”. If this instruction is not present, then do not apply the penalty.

Example: A financial question demands accuracy correct to 2dp.
Prior to rounding the answer is \$231.6189

Markscheme	Candidate's Script	Marking
<p>Financial accuracy penalty (<i>FP</i>) is applicable in this question</p> <p>\$231.62 (<i>AI</i>)</p>	<p>\$231.62 or 231.62</p> <p>231.6 or 232</p> <p>231.61</p> <p>231</p> <p>232.00</p>	<p>(<i>AI</i>) (No unit penalty (see section 7 below) for missing \$ symbol.)</p> <p>(<i>A0</i>)(<i>FP</i>) (Correct rounding process but incorrect level.)</p> <p>(<i>A0</i>)(<i>AP</i>) (Incorrect rounding process to correct level.)</p> <p>(<i>A0</i>)(<i>FP</i>) (Both types of error occurred but (<i>FP</i>) takes priority.)</p> <p>(<i>A0</i>)(<i>AP</i>) (It's not clear whether nearest dollar or 2dp was really intended but we interpret as 2dp rounded incorrectly.)</p>

7 Units in answers

A penalty known as a UNIT PENALTY (*UP*) is applied if an answer does not include the correct units. This applies both to missing units and to incorrect units. This penalty is applied to the final answer of a question part only.

THE UNIT PENALTY IS APPLIED AT MOST ONCE PER PAPER! Subsequent unit errors can be ignored and full marks awarded if all else is correct.

A unit penalty must be recorded in proximity to the incorrect answer as (*A0*)(*UP*). Examiners must record the occurrence of a unit penalty by writing (*UP*) next to the relevant question total on the front cover sheet.

NOTE: The unit penalty will be flagged in the markscheme at the start of each answer where it could apply, with the words “Unit penalty (*UP*) is applicable in this question”. If this instruction is not present, then do not apply the penalty.

NOTE: In this context, symbols for currency such as \$ or GBP etc are not considered units. Candidates are encouraged to include them but should not be penalised if they are missing.

Missing degree symbols and percentage symbols are also not eligible for a unit penalty.

No single answer can receive two penalties. If an answer is rounded incorrectly and also has wrong or missing units, apply the accuracy penalty (*AP*) only. If the (*AP*) has already been used, such an answer is eligible for the unit penalty.

Example: A question has answer to part (i) of 66.2 cm. The answer before rounding is 66.213 cm. Part (ii) involves dividing by 60 with units cm/s. Assume that the (*UP*) has not been used previously.

Markscheme	Candidate’s Script	Marking
<i>Unit penalty (UP) is applicable in this question</i>		
(i) 66.2 cm (AI)	(i) 66.2cm	(AI)
(ii) 1.10 cm/s (AI)	(ii) 1.10 cm/s	(AI)
	(i) 66.2	(A0)(UP)
	(ii) 1.10	(AI)
	(i) 66.2 cm	(AI)
	(ii) 1.10	(A0)(UP)
	(i) 66	(A0)(AP) if (AP) not used previously but (A0)(UP) otherwise.
	(ii) 1.1	(A0)(UP) if (AP) used in part (i) but (AI)(ft) for correct

	(i) 66	follow through to exact answer if (UP) used in part (i).
	(ii) 1.1 cm/s	(A0)(AP) if (AP) not used previously but (A0)(UP) otherwise. (AI)(ft)

8 Graphic Display Calculators

Candidates will often be obtaining solutions directly from their calculators. They must use mathematical notation, not calculator notation. No method marks can be awarded for incorrect answers supported only by calculator notation. The comment 'I used my GDC' cannot receive a method mark.

<p>Q1</p>	<p><i>Unit penalty (UP) is applicable where indicated in the left hand column.</i></p> <p>(a) Total = 2+3+5+7+11+5+6+9+2+1 <i>(M1) is for a sum of frequencies.</i> = 51</p> <p>(b) (i) modal interval is 60 – 70 <i>Award (A0) for 65</i></p> <p>(ii) median is length of fish no. 26, also 60 – 70 <i>Can award (A1)(ft) or (G2)(ft) for 65 if (A0) was awarded for 65 in part (i).</i></p> <p>(iii) mean is $\frac{2 \times 25 + 3 \times 35 + 5 \times 45 + 7 \times 55 + \dots}{51}$ = 69.5 cm (3sf) <i>Note: (M1) is for a sum of (frequencies multiplied by midpoint values) divided by candidate's answer from part (a). Accept mid-points 25.5, 35.5 etc or 24.5, 34.5 etc, leading to answers 70.0 or 69.0 (3sf) respectively. Answers of 69.0, 69.5 or 70.0 (3sf) with no working can be awarded (G1).</i></p> <p>(UP) (c) (i) standard deviation is 21.8 cm <i>For any other answer without working, award (G0). If working is present then (G0)(AP) is possible.</i></p> <p>(ii) $69.5 + 3 \times 21.8 = 134.9 > 120$ no fish <i>For 'no fish' without working, award (G1) regardless of answer to (c)(i). Follow through from (c)(i) only if method is shown.</i></p> <p>(d) 5 fish are less than 40 cm in length, <i>Award (M1) for any of $\frac{5}{51}, \frac{46}{51}, 0.098$ or 9.8%, 0.902, 90.2% or 5.1 seen.</i> hence no fine.</p> <p><i>Note: There is no G mark here and (M0)(A1) is never allowed. The follow-through is from answer in part (a).</i></p> <p>(e) (i) and (iii) are correct.</p>	<p>(M1) (A1)(G2) (A1) (M1) (A1)(G2) (M1) (A1)(ft)(G1) (G1) (M1) (A1)(ft)(G1) (M1) (A1)(ft) (A1)(A1)</p>	<p>[2 marks] [5 marks] [3 marks] [2 marks] [2 marks] Total [14 marks]</p>
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<p>Q2</p>	<p><i>Unit penalty (UP) is applicable where indicated in the left hand column.</i></p> <p><i>See instructions at the end if radians are used.</i></p> <p>(i) (a) $\pi \times 3.25^2 \times 39$ (= 1294.1398)</p> <p>Answer 1294.14 (cm³)(2dp)</p> <p><i>(UP) not applicable in this part due to wording of question. (MI) is for substituting appropriate numbers from the problem into the correct formula, even if the units are mixed up. (AI) is for correct substitutions or correct answer with more than 2dp in cubic centimetres seen.</i></p> <p><i>Award (GI) for answer to > 2dp with no working and no attempt to correct to 2dp.</i></p> <p><i>Award (MI)(A0)(AI)(ft) for $\pi \times 32.5^2 \times 39 \text{ cm}^3$ (= 129413.9824) = 129413.98</i></p> <p><i>Use of $\pi = \frac{22}{7}$ or 3.142 etc is premature rounding and is awarded at most (MI)(AI)(A0) or (MI)(A0)(AI)(ft) depending on whether the intermediate value is seen or not. For all other incorrect substitutions, award (MI)(A0) and only follow through the 2 dp correction if the intermediate answer to more decimal places is seen.</i></p> <p><i>Answer given as a multiple of π is awarded at most (MI)(AI)(A0).</i></p> <p><i>As usual, an unsubstituted formula followed by correct answer only receives the G marks.</i></p> <p>(b) $39/6.5 = 6$</p> <p>(UP) (c) (i) Volume of one ball is $\frac{4}{3}\pi \times 3.25^3 \text{ cm}^3$</p> <p>Volume of air = $\pi \times 3.25^2 \times 39 - 6 \times \frac{4}{3}\pi \times 3.25^3 = 431 \text{ cm}^3$</p> <p><i>Award first (MI) for substituted volume of sphere formula or for numerical value of sphere volume seen (143.79... or 45.77... $\times \pi$).</i></p> <p><i>Award second (MI) for subtracting candidate's sphere volume multiplied by their answer to (b).</i></p> <p><i>Follow through from parts (a) and (b) only, but negative or zero answer is always awarded (A0)(ft)</i></p> <p>(UP) (ii) 0.000431 m^3 or $4.31 \times 10^{-4} \text{ m}^3$</p>	<p>(MI)(AI)</p> <p>(AI)(ft)(G2)</p> <p>(AI)</p> <p>(MI)</p> <p>(MI)(AI)(ft)(G2)</p> <p>(AI)(ft)</p>	<p>[3 marks]</p> <p>[1 mark]</p> <p>[4 marks]</p>
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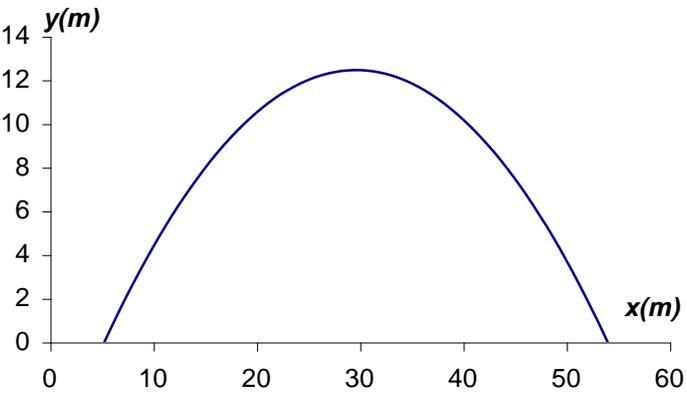
Question 2 continued

(UP)	<p>(ii) (a) (i) Angle $\widehat{BTL} = 180 - 80 - 26.5$ or $180 - 90 - 26.5 + 10 = 73.5^\circ$</p> <p>(ii) $\frac{BT}{\sin(26.5^\circ)} = \frac{120}{\sin(73.5^\circ)}$ $BT = 55.8 \text{ m (3sf)}$</p>	<p>(M1) (A1)(G2)</p> <p>(M1)(A1)(ft) (A1)(ft)</p>	<p>[5 marks]</p>
(UP)	<p>(b) $TG = 55.8\sin(80^\circ)$ or $55.8\cos(10^\circ)$ $= 55.0 \text{ m (3sf)}$ <i>Apply (AP) if 0 missing</i></p>	<p>(M1) (A1)(ft)(G2)</p>	<p>[2 marks]</p>
(UP)	<p>(c) $MT^2 = 200^2 + 55.8^2 - 2 \times 200 \times 55.8 \times \cos(100^\circ)$ $MT = 217 \text{ m (3sf)}$</p> <p><i>Follow through only from part (ii)(a)(ii). Award marks at discretion for any valid alternative method.</i></p> <p><i>If radian mode has been used throughout the question, award (A0) to the first incorrect answer then follow through, but negative lengths are always awarded (A0)(ft). The answers are (all 3sf)</i></p> <p>(a)(ii) -124 m (A0)(ft) (b) 123 m (A1)(ft) (c) 313 m (A1)(ft)</p>	<p>(M1)(A1)(ft) (A1)(ft)</p>	<p>[3 marks]</p> <p>Total [18 marks]</p>

<p>Q3</p>	<p><i>Unit penalty (UP) is applicable where indicated in the left hand column.</i></p> <p>(i) (a) equation of asymptote is $x = 0$ (Must be an equation.)</p> <p>(b) $f'(x) = 2x + 3x^{-2}$ (or equivalent) (AI) for each term</p> <p>(c) stationary point $(-1.14, 3.93)$ $(-1, 4)$ or similar error is awarded (G0)(GI)(ft) Here and also as follow through in part (d) accept exact values $-\left(\frac{3}{2}\right)^{\frac{1}{3}}$ for the x coordinate and $3\left(\frac{3}{2}\right)^{\frac{2}{3}}$ for the y coordinate.</p> <p>OR $2x + \frac{3}{x^2} = 0$ or equivalent Correct coordinates as above Follow through from candidate's $f'(x)$</p> <p>(d) In all alternative answers for (d), follow through from candidate's x coordinate in part (c). Alternative answers include: $-1.14 \leq x < 0, \quad 0 < x < 5$</p> <p>OR $[-1.14, 0), (0, 5)$ Accept alternative bracket notation for open interval] [. (Union of these sets is not correct, award (A2) if all else is right in this case.)</p> <p>OR $-1.14 \leq x < 5, x \neq 0$</p> <p><i>In all versions 0 must be excluded (AI) -1.14 must be the left bound (AI)(ft) 5 must be the right bound (AI). For $x \geq -1.14$ or $x > -1.14$ alone, award (AI). For $-1.4 \leq x < 0$ together with $x > 0$ award (A2).</i></p>	<p>(AI)</p> <p>(AI)(AI)</p> <p>(GI)(GI)(ft)</p> <p>(MI)</p> <p>(AI)(ft)</p> <p>(AI)(AI)(ft)</p> <p>(AI)</p>	<p>[1 mark]</p> <p>[2 marks]</p> <p>[2 marks]</p> <p>[3 marks]</p>
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Question 3 continued

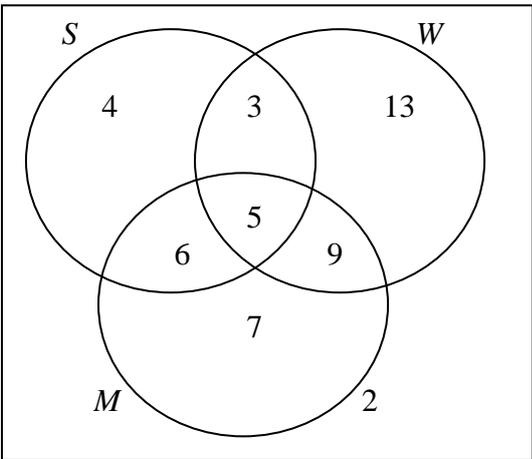
	<p>(ii) (a) $a = 5.30$ (3sf) (Allow (5.30, 0) but 5.3 receives an (AP).)</p> <p>(b) $\frac{dy}{dx} = -0.042x + 1.245$ (AI) for each term.</p> <p>(c) (i) Maximum value when $f'(x) = 0$, $-0.042x + 1.245 = 0$, (MI) is for either of the above but at least one must be seen. ($x = 29.6$) Football has travelled $29.6 - 5.30 = 24.3$ m (3sf) horizontally. For answer of 24.3 m with no working or for correct subtraction of 5.3 from candidate's x-coordinate at the maximum (if not 29.6), award (AI)(d).</p>	<p>(AI)</p> <p>(AI)(AI)</p> <p>(MI)</p> <p>(AI)(ft)</p>	<p>[1 mark]</p> <p>[2 marks]</p>
(UP)	<p>(ii) Maximum vertical height, $f(29.6) = 12.4$ m (MI) is for substitution into f of a value seen in part (c)(i). $f(24.3)$ with or without evaluation is awarded (MI)(A0). For any other value without working, award (G0). If lines are seen on the graph in part (d) award (MI) and then (AI) for candidate's value ± 0.5 (3sf not required.)</p>	<p>(MI)(AI)(ft)(G2)</p>	<p>[4 marks]</p>
	<p>(d) (not to scale)</p>  <p>Award (AI) for labels (units not required) and scale, (AI)(ft) for max (29.6, 12.4), (AI)(ft) for x-intercepts at 5.30 and 53.9, (all coordinates can be within 0.5), (AI) for well-drawn parabola ending at the x-intercepts.</p>	<p>(AI)(AI)</p> <p>(AI)(ft)</p> <p>(AI)(ft)</p>	
(UP)	<p>(e) $f(40.3) = 10.1$ m (3sf). Follow through from (a). If graph used, award (MI) for lines drawn and (AI) for candidate's value ± 0.5. (3sf not required).</p>	<p>(MI)(AI)(ft)(G2)</p>	<p>[4 marks]</p> <p>[2 marks]</p>
			<p>Total [21 marks]</p>

Q4	(i) (a)	$u_1 = d = 1.$	(AI)(AI)	[2 marks]
	(b)	Sum is $\frac{1}{2}n(2u_1 + d(n-1))$ or $\frac{1}{2}n(u_1 + u_n)$ Award (MI) for either sum formula seen, even without substitution. So sum is $\frac{1}{2}n(2 + (n-1)) = \frac{1}{2}n(n+1)$ Award (AI) for substitution of $u_1 = 1 = d$ or $u_1 = 1$ and $u_n = n$ with simplification where appropriate. $\frac{1}{2}n(n+1)$ must be seen to award this (AI).	(MI) (AI)(AG)	[2 marks]
	(c)	$\frac{1}{2}(200)(201) = 20\ 100$ (MI) is for correct formula with correct numerical input. Original sum formula with u , d and n can be used.	(MI)(AI)(G2)	[2 marks]
	(ii) (a)	$\frac{1-3^n}{1-3} = 29\ 524$ (MI) for correctly substituted formula on one side, (AI) for = 29524 on the other side. $n = 10.$ Trial and error is a valid method. Award (MI) for at least $\frac{1-3^{10}}{1-3}$ seen and then (AI) for = 29524, (AI) for $n = 10.$ For only unproductive trials with $n \neq 10$, award (MI) and then (AI) if the evaluation is correct.	(MI)(AI) (AI)(G2)	[3 marks]
	(b)	Common ratio is $\frac{1}{3}$, (0.333 (3sf) or $0.\dot{3}$) Accept 'divide by 3'.	(AI)	[1 mark]
	(c)	$\frac{1 - \left(\frac{1}{3}\right)^{10}}{1 - \frac{1}{3}}$ = 1.50 (3sf) 1.5 and $\frac{3}{2}$ receive (A0)(AP) if AP not yet used Incorrect formula seen in (a) or incorrect value in (b) can follow through to (c). Can award (MI) for $1 + \left(\frac{1}{3}\right) + \left(\frac{1}{9}\right) + \dots$	(MI) (AI)(ft)(GI)	[2 marks]

<p>Q5</p>	<p>(i) (a)</p>	<p>(i) P (a dog is grey and has the yellow bowl) $= \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$ (=0.111) <i>The (MI) is for multiplying two values along any branch of the tree.</i></p>	<p>(MI)(AI)(G2)</p>	<p>[3 marks]</p>
		<p>(ii) P (dog does not get yellow bowl) = $\frac{2}{3}$ (= 0.667 (3sf) or 0.6)</p>	<p>(AI)</p>	
		<p>(b) (i) The tree diagram should show the values $\frac{1}{2}, \frac{1}{2}$ for the brown branch and $\frac{1}{4}, \frac{3}{4}$ in the correct positions for the grey branch. <i>Follow through if the branches are interchanged.</i></p>	<p>(AI) (AI)(ft)</p>	
		<p>(ii) P (the dog is grey or is playing with a stick, but not both) $= \frac{1}{3} \times \frac{3}{4} + \frac{2}{3} \times \frac{1}{2}$ $= \frac{7}{12}$ (= 0.583) <i>The (MI) is for showing two correct products (whether added or not). Follow through from b(i). Award (MI) for $\frac{1}{3} + \frac{1}{4}$ (must be a sum).</i></p>	<p>(MI) (AI)(ft)(GI)</p>	
		<p>(iii) P (dog is grey given that it is playing with stick) $\frac{P(G \cap S)}{P(S)} = \frac{\frac{1}{3} \times \frac{1}{4}}{\left(\frac{2}{3} \times \frac{1}{2}\right) + \left(\frac{1}{3} \times \frac{1}{4}\right)} \text{ or } \frac{1}{12} / \frac{5}{12}$ <i>(MI) for substituted conditional probability formula, (AI) for correct substitutions.</i> $= \frac{1}{5}$ (= 0.2)</p>	<p>(MI)(AI)(ft) (AI)(ft)(G2)</p>	
		<p>(iv) P (grey and fed from yellow bowl and not playing with stick) = $\frac{1}{3} \times \frac{1}{3} \times \frac{3}{4} = \frac{1}{12}$ (= $\frac{3}{36}$ = 0.0833 3sf). <i>(MI) is for product of 3 reasonable probability values.</i></p>	<p>(MI) (AI)(ft)(GI)</p>	

continued...

Question 5 continued

<p>(ii) (a)</p>  <p>Award (A1) for 2 (must be in a box), (A1) for 7, (A1)(ft) for 6 and 4, (A1)(ft) for 9 and 13 Observe the assignment of (ft) marks strictly here. Example A common error is likely to be 11 instead of 6 (A0). In this case follow through to 4 and 18 (A1)(ft) for the final pair. Here the 4 follows from the total of 27 for $n(M)$.</p> <p>(b) (i) $n(M \cap W) = 14$</p> <p>(ii) $n(M' \cup S) = 22+11$ OR $15+18$ $= 33$ Award (A2) if answer 33 is seen. Award (A1) for any of 22, 11, 15 or 18 seen but 33 absent.</p> <p>(c) $P(\text{both mice short-tailed}) = \frac{18}{49} \times \frac{17}{48} = \frac{306}{2352} (= 0.130)$. (Allow alternatives such as $153/1176$ or $51/392$.) Award (M1) for any of $\frac{18}{49}$ and $\frac{17}{48}$ or $\frac{18}{49} \times \frac{17}{49}$ or $\frac{18}{49} + \frac{17}{48}$ seen.</p>	<p>(A1)(A1) (A1)(ft) (A1)(ft)</p> <p>(A1)(ft)</p> <p>(A1)(ft) (A1)(ft)</p> <p>(M1)(A1)(ft) (G1)</p>	<p>[4 marks]</p> <p>[3 marks]</p> <p>[2 marks]</p> <p>Total [21 marks]</p>
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